VIF Vienna Institute of Finance

Working Paper Series

Working Paper No. 17

Stability Results for Term Structure Models driven by Lévy processes

Barbara Rüdiger and Stefan Tappe

First version: May 2009

Current version: May 2009

WWTF

WIRTSCHAFTS UNIVERSITÄT WIEN

universität wien

STABILITY RESULTS FOR TERM STRUCTURE MODELS DRIVEN BY LÉVY PROCESSES

BARBARA RÜDIGER AND STEFAN TAPPE

Abstract. We provide stability results for term structure models driven by Lévy processes. Regarding such a model as the solution of a stochastic partial differential equation, the so-called HJMM equation, we prove stability with respect to perturbations of the volatilities and the initial forward curve. We also study regular dependence on initial data and show, for a differentiable curve of initial data, convergence to the first variation process.

Key Words: Lévy term structure models, stability, regular dependence on initial data, stochastic partial differential equations.

91B28, 60H15

1. Introduction

In reality, one can never be sure of the accuracy of a proposed model for financial market fluctuations or other random phenomena. The natural question arises, if one perturbs the model a bit, how large a the resulting changes? In this text, we investigate stability of Lévy term structure models. More precisely, we consider a market of zero coupon bonds

$$P(t,T) = \exp\left(- \int_t^{T-t} r_t(x)dx \right), \quad 0 \leq t \leq T \tag{1.1}$$

where the forward rates $r_t(x)$ (in the Musiela parametrization [25]) are driven by a Lévy process $X = (X^1, \dots, X^d)$. Such models, which generalize the classical Heath, Jarrow, Morton (HJM) model [19] driven by a Wiener process, have been proposed by Eberlein et al. [7, 8, 9, 10, 11, 12]. Other approaches in order to generalize the classical HJM framework can be found in Björk et al. [2, 3], Carmona and Tehranchi [4], and, e.g., [29, 21, 20].

Since, from a financial modeling point of view, one would like to incorporate the current state of the forward curve, it was suggested to model the forward curves as the solution of a stochastic partial differential equation (SPDE), the so-called HJMM (Heath–Jarrow–Morton–Musiela) equation

$$\begin{cases} dr_t &= (\frac{d}{dx} r_t + \alpha_{\text{HJM}}(r_t))dt + \sum_{j=1}^d \sigma^j(r_{t-})dX_t^j \\ r_0 &= h_0 \end{cases} \tag{1.2}$$

on a suitable Hilbert space H of forward curves, where $\frac{d}{dx}$ denotes the differential operator, which is generated by the strongly continuous semigroup $(S_t)_{t \geq 0}$ of shifts.

For term structure models driven by a Brownian motion, the existence proof has been provided in [13], and for the Lévy case, which we consider in this text, in [14]. We also refer to the related papers [27] and [23].

Date: May 12, 2009.

The second author gratefully acknowledges the support from WWTF (Vienna Science and Technology Fund).

We are grateful to Damir Filipović, Vidyadhar Mandrekar and Josef Teichmann for their helpful remarks and discussions.

The bond market (1.1) is free of arbitrage if we can find an equivalent (local) martingale measure such that discounted bond prices

$$\exp\bigg(- \int_0^t r_s(0) ds \bigg) P(t,T), \quad t \in [0,T]$$

are local martingales for all maturities T. If we formulate the HJMM equation (1.2) with respect to such an equivalent martingale measure, then the drift is determined by the volatilities, i.e. $\alpha_{\text{HJM}} : H \to H$ is given by the HJM drift condition

$$\alpha_{\text{HJM}}(h) := - \sum_{j=1}^d \sigma^j(h) \Psi_j' \bigg(- \int_0^\bullet \sigma^j(h)(\eta) d\eta \bigg), \quad h \in H \tag{1.3}$$

where the Ψ_j denote the cumulant generating functions of the Lévy processes, see [8, Sec. 2.1].

In this text, we show that Lévy term structure models of the type (1.2) are stable with respect to perturbations of the volatilities σ^j and the initial forward curve h_0. We also study regular dependence on initial data and show that, for a differentiable curve $\epsilon \mapsto c(\epsilon)$ of initial data with $c'(0) = w$ we obtain convergence to the first variation process $J(r) \bullet w$ (see Appendix C for the definition of the first variation process).

In the recent paper [15] a new approach to stochastic partial differential equations, called the method of the moving frame, was suggested. This approach allows to reduce a wide class of SPDE problems to SDEs and admits stability results, which are suitable for this text. However, for an application of those results to Lévy term structure models, we have, due to the absence of arbitrage, to take into account the particular structure of the HJM drift term α_{HJM} from (1.3).

Finally, we mention that, recently, the existence proof in the more general situation, where, in the spirit of [2], the HJMM equation is driven by a (possibly infinite dimensional) Wiener process and a compensated Poisson random measure, was established in [16]. However, then the structure of the corresponding HJM drift α_{HJM} becomes quite involved, whence we focus our attention on the Lévy case here.

The remainder of this text is organized as follows. In Section 2 we introduce the space H_β of forward curves. Using this space, we establish stability of Lévy term structure models in Section 3 and regular dependence on initial data for Lévy term structure models in Section 4. The required results for general stochastic partial differential equations are provided in Appendices A, B, C.

2. The space of forward curves

In this section, we define the space of forward curves, on which we will study the HJMM equation (1.2) in the forthcoming sections. These spaces have been introduced in [13, Sec. 5].

We fix an arbitrary constant $\beta > 0$. Let H_β be the space of all absolutely continuous functions $h : \mathbb{R}_+ \to \mathbb{R}$ such that

$$\|h\|_\beta := \bigg(|h(0)|^2 + \int_{\mathbb{R}_+} |h'(x)|^2 e^{\beta x} dx \bigg)^{\frac{1}{2}} < \infty.$$

Let $(S_t)_{t \geq 0}$ be the shift semigroup on H_β defined by $S_t h := h(t + \cdot)$ for $t \in \mathbb{R}_+$.

Since forward curves should flatten for large time to maturity x, the choice of H_β is reasonable from an economic point of view.

2.1. **Theorem.** *Let $\beta > 0$ be arbitrary.*

(1) *The space $(H_\beta, \| \cdot \|_\beta)$ is a separable Hilbert space.*

(2) *For each* $x \in \mathbb{R}_+$, *the point evaluation* $h \mapsto h(x) : H_\beta \to \mathbb{R}$ *is a continuous linear functional.*

(3) $(S_t)_{t\geq 0}$ *is a* C_0*-semigroup on* H_β *with infinitesimal generator* $\frac{d}{dx} : \mathcal{D}(\frac{d}{dx}) \subset H_\beta \to H_\beta$, $\frac{d}{dx}h = h'$, *and domain*

$$\mathcal{D}(\tfrac{d}{dx}) = \{h \in H_\beta \,|\, h' \in H_\beta\}.$$

(4) *Each* $h \in H_\beta$ *is continuous, bounded and the limit* $h(\infty) := \lim_{x\to\infty} h(x)$ *exists.*

(5) $H_\beta^0 := \{h \in H_\beta \,|\, h(\infty) = 0\}$ *is a closed subspace of* H_β.

(6) *There exists a universal constant* $C > 0$, *only depending on* β, *such that for all* $h \in H_\beta$ *we have the estimate*

$$\|h\|_{L^\infty(\mathbb{R}_+)} \leq C\|h\|_\beta, \tag{2.1}$$

(7) *For each* $\beta' > \beta$, *we have* $H_{\beta'} \subset H_\beta$ *and the relation*

$$\|h\|_\beta \leq \|h\|_{\beta'}, \quad h \in H_{\beta'} \tag{2.2}$$

(8) *There exist another separable Hilbert space* $\mathcal{H}_\beta$, *a* C_0*-group* $(U_t)_{t\in\mathbb{R}}$ *on* $\mathcal{H}_\beta$ *and continuous linear operators* $\ell \in L(H_\beta, \mathcal{H}_\beta)$, $\pi \in L(\mathcal{H}_\beta, H_\beta)$ *such that the diagram*

$$\begin{array}{ccc} \mathcal{H}_\beta & \xrightarrow{U_t} & \mathcal{H}_\beta \\ \uparrow \ell & & \downarrow \pi \\ H_\beta & \xrightarrow{S_t} & H_\beta \end{array}$$

commutes for every $t \in \mathbb{R}_+$, *that is*

$$\pi U_t \ell h = S_t h \quad \text{for all } t \in \mathbb{R}_+ \text{ and } h \in H_\beta.$$

Proof. See [16, Thm. 2.1]. □

Note that the latter statement of Theorem 2.1 ensures that Assumption A.4 from Appendix A is fulfilled, whence, in the sequel, we may apply the results obtained by the method of the moving frame in [15]. Actually, the particular structure of the Hilbert space $\mathcal{H}_\beta$ is not relevant for this text. For the sake of completeness, we shortly review its definition. It is constructed in a natural way, namely, let $\mathcal{H}_\beta$ be the space of all absolutely continuous functions $h : \mathbb{R} \to \mathbb{R}$ such that

$$\|h\|_\beta := \left(|h(0)|^2 + \int_{\mathbb{R}} |h'(x)|^2 e^{\beta|x|} dx \right)^{\frac{1}{2}} < \infty.$$

Let $(U_t)_{t\in\mathbb{R}}$ be the shift group on $\mathcal{H}_\beta$ defined by $U_t h := h(t + \cdot)$ for $t \in \mathbb{R}$. Furthermore, we define the isometric embedding $\ell : H_\beta \to \mathcal{H}_\beta$ as

$$\ell(h)(x) := \begin{cases} h(0), & x < 0 \\ h(x), & x \geq 0, \end{cases} \qquad h \in H_\beta$$

and let $\pi := \ell^* : \mathcal{H}_\beta \to H_\beta$ be its adjoint operator, which is given by $\pi(h) = h|_{\mathbb{R}_+}$, $h \in \mathcal{H}_\beta$.

3. Stability of Lévy term structure models

In this section we provide the announced stability result, showing that Lévy term structure models are stable with respect to perturbations of the volatilities and the initial forward curve.

Throughout this text, let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ be a filtered probability space satisfying the usual conditions. Let $d, e \in \mathbb{N}_0$ be integers such that $d + e \geq 1$. We denote by $W^1, \dots, W^d$ real-valued, independent Wiener processes, and by $X^1, \dots, X^e$ real-valued, independent Lévy martingales without diffusion part, which have the Lévy measures $F_1, \dots, F_e$.

3.1. **Assumption.** *We assume there exist constants $N, \epsilon > 0$ such that for all $k = 1, \dots, e$ we have*

$$\int_{\{|x|>1\}} e^{zx} F_k(dx) < \infty, \quad z \in [-(1+\epsilon)N, (1+\epsilon)N]$$

Then, the cumulant generating functions

$$\Psi_k(z) := \ln \mathbb{E}[e^{zX_1^k}], \quad k = 1, \dots, e$$

exist on $[-N, N]$ and are of class C^∞, see [28, Lemma 26.4]. In particular, there exists a constant $K > 0$ such that for all $k = 1, \dots, e$ we have

$$|\Psi_k^{(n)}(x)| \leq K, \quad x \in [-N, N] \text{ and } n = 0, \dots, 4. \tag{3.1}$$

Let $\beta > 0$ be an arbitrary real number. Let volatilities $\sigma^j : H_\beta \to H_\beta^0$, $j = 1, \dots, d$, $\delta^k : H_\beta \to H_\beta^0$, $k = 1, \dots, e$ and $\sigma_n^j : H_\beta \to H_\beta^0$, $j = 1, \dots, d$, $\delta_n^k : H_\beta \to H_\beta^0$, $k = 1, \dots, e$ for each $n \in \mathbb{N}$ be given.

3.2. **Assumption.** *We assume that for all $h \in H_\beta$, $x \in \mathbb{R}_+$ and $k = 1, \dots, e$ we have*

$$\left| \int_0^x \delta^k(h)(\eta) d\eta \right| \leq N,$$

$$\left| \int_0^x \delta_n^k(h)(\eta) d\eta \right| \leq N, \quad n \in \mathbb{N}.$$

According to the HJM drift condition (1.3) we define

$$\alpha_{\mathrm{HJM}}(h) := \sum_{j=1}^d \sigma^j(h) \int_0^\bullet \sigma^j(h)(\eta) d\eta - \sum_{k=1}^e \delta^k(h) \Psi_k' \Big(- \int_0^\bullet \delta^k(h)(\eta) d\eta \Big) \tag{3.2}$$

$$\alpha_{\mathrm{HJM}}^n(h) := \sum_{j=1}^d \sigma_n^j(h) \int_0^\bullet \sigma_n^j(h)(\eta) d\eta - \sum_{k=1}^e \delta_n^k(h) \Psi_k' \Big(- \int_0^\bullet \delta_n^k(h)(\eta) d\eta \Big), \quad n \in \mathbb{N} \tag{3.3}$$

for each $h \in H_\beta$.

3.3. **Assumption.** *We assume that for all $h \in H_\beta$ we have $\sigma_n^j(h) \to \sigma^j(h)$, $j = 1, \dots, d$ and $\delta_n^k(h) \to \delta^k(h)$, $k = 1, \dots, e$.*

3.4. **Assumption.** *We assume there exist a constant $L > 0$ such that*

$$\|\sigma^j(h_1) - \sigma^j(h_2)\|_\beta \leq L \|h_1 - h_2\|_\beta, \quad j = 1, \dots, d \tag{3.4}$$

$$\|\delta^k(h_1) - \delta^k(h_2)\|_\beta \leq L \|h_1 - h_2\|_\beta, \quad k = 1, \dots, e \tag{3.5}$$

$$\|\sigma_n^j(h_1) - \sigma_n^j(h_2)\|_\beta \leq L \|h_1 - h_2\|_\beta, \quad j = 1, \dots, d \text{ and } n \in \mathbb{N} \tag{3.6}$$

$$\|\delta_n^k(h_1) - \delta_n^k(h_2)\|_\beta \leq L \|h_1 - h_2\|_\beta, \quad k = 1, \dots, e \text{ and } n \in \mathbb{N} \tag{3.7}$$

for all $h_1, h_2 \in H_\beta$, and a constant $M > 0$ such that

$$\|\sigma^j(h)\|_\beta \leq M, \quad j = 1, \dots, d \tag{3.8}$$

$$\|\delta^k(h)\|_\beta \leq M, \quad k = 1, \dots, e \tag{3.9}$$

$$\|\sigma_n^j(h)\|_\beta \leq M, \quad j = 1, \dots, d \text{ and } n \in \mathbb{N} \tag{3.10}$$

$$\|\delta_n^k(h)\|_\beta \leq M, \quad k = 1, \dots, e \text{ and } n \in \mathbb{N} \tag{3.11}$$

for all $h \in H_\beta$.

3.5. **Proposition.** *Suppose Assumptions 3.1, 3.2, 3.3, 3.4 are fulfilled. Then we have* $\alpha_{\mathrm{HJM}}(H_\beta) \subset H_\beta^0$, $\alpha_{\mathrm{HJM}}^n(H_\beta) \subset H_\beta^0$ *for all* $n \in \mathbb{N}$ *and* $\alpha_{\mathrm{HJM}}^n(h) \to \alpha_{\mathrm{HJM}}(h)$ *for each* $h \in H_\beta$. *Moreover, there exists a constant* $\tilde{L} > 0$ *such that*

$$\|\alpha_{\mathrm{HJM}}(h_1) - \alpha_{\mathrm{HJM}}(h_2)\|_\beta \leq \tilde{L}\|h_1 - h_2\|_\beta, \tag{3.12}$$

$$\|\alpha_{\mathrm{HJM}}^n(h_1) - \alpha_{\mathrm{HJM}}^n(h_2)\|_\beta \leq \tilde{L}\|h_1 - h_2\|_\beta, \quad n \in \mathbb{N} \tag{3.13}$$

for all $h_1, h_2 \in H_\beta$.

Proof. The claim follows from [13, Cor. 5.1.2] and [14, Prop. 4.5]. □

Now let arbitrary initial curves $h_0 \in L^2(\Omega, \mathcal{F}_0, \mathbb{P}; H_\beta)$ and $h_0^n \in L^2(\Omega, \mathcal{F}_0, \mathbb{P}; H_\beta)$, $n \in \mathbb{N}$ be given. Let $(r_t)_{t \geq 0}$ be the mild solution for the HJMM equation

$$\begin{cases} dr_t = (\frac{d}{dx} r_t + \alpha_{\mathrm{HJM}}(r_t))dt + \sum_{j=1}^d \sigma^j(r_t) dW_t^j + \sum_{k=1}^e \delta^k(r_{t-}) dX_t^k \\ r_0 = h_0, \end{cases} \tag{3.14}$$

and for each $n \in \mathbb{N}$ let $(r_t^n)_{t \geq 0}$ be the mild solution for the HJMM equation

$$\begin{cases} dr_t^n = (\frac{d}{dx} r_t^n + \alpha_{\mathrm{HJM}}^n(r_t^n))dt + \sum_{j=1}^d \sigma_n^j(r_t^n) dW_t^j + \sum_{k=1}^e \delta_n^k(r_{t-}^n) dX_t^k \\ r_0^n = h_0^n \end{cases}$$

on the state space H_β. We require one further assumption.

3.6. **Assumption.** *We assume that* $h_0^n \to h_0$ *in* $L^2(\Omega, \mathcal{F}_0, \mathbb{P}; H)$.

Here is our result on the stability of Lévy term structure models.

3.7. **Theorem.** *Suppose Assumptions 3.1, 3.2, 3.3, 3.4, 3.6 are fulfilled. Then, for every* $T \in \mathbb{R}_+$ *we have*

$$\mathbb{E}\left[\sup_{t \in [0,T]} \|r_t - r_t^n\|_\beta^2\right] \to 0$$

for $n \to \infty$.

Proof. Taking into account Theorem 2.1 and Proposition 3.5, the assertion follows from Theorem B.4. □

4. Regular dependence on initial data for Lévy term structure models

In this section, we study regular dependence on initial data for Lévy term structure models and show, for a differentiable curve of initial data, convergence to the first variation process.

As in the previous section, we suppose that Assumption 3.1 is fulfilled. In order to derive the differentiability of the HJM drift term (3.2), we prepare some auxiliary results.

For any normed space $(X, \|\cdot\|)$ we let $B_X := \{x \in X : \|x\| \leq 1\}$ be the closed unit ball.

Let $0 < \beta < \beta'$ be arbitrary real numbers. We define $\mathcal{I}h := \int_0^\bullet h(\eta) d\eta$ for all $h \in H_{\beta'}^0$.

4.1. **Lemma.** *For each* $h \in H_{\beta'}^0$ *we have* $\mathcal{I}h \in H_\beta$ *and the map* $\mathcal{I} : H_{\beta'}^0 \to H_\beta$ *is a continuous linear operator with operator norm* $\|\mathcal{I}\| \leq \sqrt{\frac{1}{\beta'(\beta' - \beta)}}$.

Proof. Let $h \in H^0_{\beta'}$ be arbitrary. Then $\mathcal{I}h$ is absolutely continuous. Since $\mathcal{I}h(0) = 0$, using Hölder's inequality, we obtain

$$\begin{aligned} \|\mathcal{I}h\|_\beta^2 &= \int_{\mathbb{R}_+} h(x)^2 e^{\beta x} dx = \int_{\mathbb{R}_+} \left(\int_x^\infty h'(y) e^{\frac{1}{2}\beta' y} e^{-\frac{1}{2}\beta' y} dy \right)^2 e^{\beta x} dx \\ &\leq \int_{\mathbb{R}_+} \left(\int_x^\infty h'(y)^2 e^{\beta' y} dy \right) \left(\int_x^\infty e^{-\beta' y} dy \right) e^{\beta x} dx \\ &\leq \|h\|_{\beta'}^2 \int_{\mathbb{R}_+} \frac{1}{\beta'} e^{-(\beta'-\beta)x} dx = \frac{1}{\beta'(\beta'-\beta)} \|h\|_{\beta'}^2, \end{aligned}$$

proving the desired statement. □

Hence, $\mathcal{I} : H^0_{\beta'} \to H_\beta$ is Fréchet differentiable. For $h \in H^0_\beta$, $g \in H_\beta$ we define the multiplication $m(h,g) := hg$.

4.2. **Lemma.** *For all $h \in H^0_\beta$, $g \in H_\beta$ we have $m(h,g) \in H^0_\beta$ and the estimate*

$$\|m(h,g)\|_\beta \leq \sqrt{C^4 + 4C^2} \|h\|_\beta \|g\|_\beta. \tag{4.1}$$

Moreover, the multiplication map $m : H^0_\beta \times H_\beta \to H^0_\beta$ is Fréchet differentiable with derivative

$$Dm(h_1,h_2) \bullet (v_1,v_2) = h_1 v_2 + h_2 v_1.$$

Proof. The function hg is again absolutely continuous with $\lim_{x\to\infty} h(x)g(x) = 0$. By estimate (2.1), we obtain

$$\begin{aligned} \|m(h,g)\|_\beta^2 &= |h(0)|^2 |g(0)|^2 + \int_{\mathbb{R}_+} |h(x)g'(x) + g(x)h'(x)|^2 e^{\beta x} dx \\ &\leq \|h\|^2_{L^\infty(\mathbb{R}_+)} \|g\|^2_{L^\infty(\mathbb{R}_+)} + 2\|h\|^2_{L^\infty(\mathbb{R}_+)} \int_{\mathbb{R}_+} |g'(x)|^2 e^{\beta x} dx \\ &\quad + 2\|g\|^2_{L^\infty(\mathbb{R}_+)} \int_{\mathbb{R}_+} |h'(x)|^2 e^{\beta x} dx \\ &\leq C^4 \|h\|_\beta^2 \|g\|_\beta^2 + 2C^2 \|h\|_\beta^2 \|g\|_\beta^2 + 2C^2 \|g\|_\beta^2 \|h\|_\beta^2 < \infty, \end{aligned}$$

whence estimate (4.1) follows, which yields

$$\begin{aligned} &\left\| \frac{m((h_1,h_2) + \epsilon(v_1,v_2)) - m(h_1,h_2)}{\epsilon} - h_1 v_2 - h_2 v_1 \right\|_\beta = \|\epsilon v_1 v_2\|_\beta \\ &\leq \sqrt{C^4 + 4C^2} \|v_1\|_\beta \|v_2\|_\beta |\epsilon| \to 0 \end{aligned}$$

for $\epsilon \to 0$ uniformly in $(v_1,v_2) \in B_{H_\beta \times H_\beta}$. □

Let $\mathcal{O} \subset H_\beta$ be the open set

$$\mathcal{O} := \{h \in H_\beta : \|h\|_\beta < \tfrac{N}{C}\},$$

By estimate (2.1) we have $\mathcal{O} \subset U$, where $U \subset H_\beta$ denotes the subset

$$U := \{h \in H_\beta : \|h\|_{L^\infty(\mathbb{R}_+)} \leq N\}.$$

4.3. **Lemma.** *Let $k \in \{1,\dots,e\}$ be arbitrary. For each $h \in U$ we have $\Psi_k'(h) \in H^0_\beta$ and the map $\Psi_k' : \mathcal{O} \to H_\beta$ is Fréchet differentiable with derivative*

$$D\Psi_k'(h) \bullet v = \Psi_k''(h) v.$$

Proof. The map $\Psi_k'(h)$ is again absolutely continuous, and, by (3.1), we have

$$\begin{aligned}\|\Psi_k'(h)\|_\beta^2 &= |\Psi_k'(h(0))|^2 + \int_{\mathbb{R}_+} |\Psi_k''(h(x))h'(x)|^2 e^{\beta x} dx \\ &\leq |\Psi_k'(h(0))|^2 + K^2 \|h\|_\beta^2 < \infty,\end{aligned}$$

whence $\Psi_k'(h) \in H_\beta$. For $\epsilon \neq 0$ small enough we obtain, by (3.1) and (2.1),

$$\begin{aligned}&\left| \frac{\Psi_k'(h(0)+\epsilon v(0)) - \Psi_k'(h(0))}{\epsilon} - \Psi_k''(h(0))v(0) \right| \\ &= \left| \int_0^1 \Big(\Psi_k''(h(0)+s\epsilon v(0))v(0) - \Psi_k''(h(0))v(0) \Big) ds \right| \\ &\leq \int_0^1 |K s \epsilon v(0)^2| ds \leq K \|v\|_{L^\infty(\mathbb{R}_+)}^2 |\epsilon| \leq K C^2 \|v\|_\beta^2 |\epsilon|.\end{aligned}$$

Thus, the latter term converges to zero for $\epsilon \to 0$, uniformly in $v \in B_{H_\beta}$. For $\epsilon \neq 0$ small enough we have, by Hölder's inequality,

$$\begin{aligned}&\int_{\mathbb{R}_+} \left| \frac{d}{dx} \left(\frac{\Psi_k'(h(x)+\epsilon v(x)) - \Psi_k'(h(x))}{\epsilon} - \Psi_k''(h(x))v(x) \right) \right|^2 e^{\beta x} dx \\ &= \int_{\mathbb{R}_+} \left| \frac{d}{dx} \left(\int_0^1 \Big(\Psi_k''(h(x)+s\epsilon v(x))v(x) - \Psi_k''(h(x))v(x) \Big) ds \right) \right|^2 e^{\beta x} dx \\ &\leq \int_{\mathbb{R}_+} \int_0^1 \left| \frac{d}{dx} \Big(\Big(\Psi_k''(h(x)+s\epsilon v(x)) - \Psi_k''(h(x)) \Big) v(x) \Big) \right|^2 e^{\beta x} ds dx \\ &\leq 2\Delta_1^\epsilon + 2\Delta_2^\epsilon,\end{aligned} \tag{4.2}$$

where we have set

$$\begin{aligned}\Delta_1^\epsilon &:= \int_{\mathbb{R}_+} \int_0^1 \Big| \Big(\Psi_k'''(h(x)+s\epsilon v(x))(h'(x)+s\epsilon v'(x)) - \Psi_k'''(h(x))h'(x) \Big) v(x) \Big|^2 \\ &\qquad \times e^{\beta x} ds dx, \\ \Delta_2^\epsilon &:= \int_{\mathbb{R}_+} \int_0^1 \Big| \Big(\Psi_k''(h(x)+s\epsilon v(x)) - \Psi_k''(h(x)) \Big) v'(x) \Big|^2 e^{\beta x} ds dx.\end{aligned}$$

By (3.1) and (2.1) we get

$$\begin{aligned}\Delta_1^\epsilon &\leq 2 \int_{\mathbb{R}_+} \int_0^1 \Big| \Big(\Psi_k'''(h(x)+s\epsilon v(x)) - \Psi_k'''(h(x)) \Big) h'(x)v(x) \Big|^2 e^{\beta x} ds dx \\ &\quad + 2 \int_{\mathbb{R}_+} \int_0^1 \big| \Psi_k'''(h(x)+s\epsilon v(x)) s\epsilon v'(x) v(x) \big|^2 e^{\beta x} ds dx \\ &\leq 2 \int_{\mathbb{R}_+} \int_0^1 |K s\epsilon v(x)^2 h'(x)|^2 e^{\beta x} ds dx + 2 \int_{\mathbb{R}_+} \int_0^1 |K s \epsilon v'(x) v(x)|^2 e^{\beta x} ds dx \\ &\leq 2K^2 \epsilon^2 C^4 \|v\|_\beta^4 \|h\|_\beta^2 + 2K^2 \epsilon^2 C^2 \|v\|_\beta^4\end{aligned}$$

as well as

$$\Delta_2^\epsilon \leq \int_{\mathbb{R}_+} \int_0^1 |K s \epsilon v(x) v'(x)|^2 e^{\beta x} ds dx \leq K^2 \epsilon^2 C^2 \|v\|_\beta^4.$$

Hence, the integral in (4.2) converges to zero for $\epsilon \to 0$ uniformly in $v \in B_{H_\beta}$. □

Now let volatilities $\sigma^j : H_\beta \to H_{\beta'}^0$, $j = 1, \dots, d$ and $\delta^k : H_\beta \to H_{\beta'}^0$, $k = 1, \dots, e$ be given.

4.4. **Assumption.** *We assume that* $-\mathcal{I}\delta^k(H_\beta) \subset \mathcal{O}$, $k = 1, \dots, e$.

Then we can define $\alpha_{\rm HJM}$ according to HJM drift condition (3.2).

4.5. **Assumption.** *We assume that* $\sigma^j(h)$, $j = 1,\ldots,d$ *and* δ^k, $k = 1,\ldots,e$ *are Fréchet differentiable.*

4.6. **Proposition.** *Suppose Assumptions 3.1, 4.4, 4.5 are fulfilled. Then we have* $\alpha_{\rm HJM}(H_\beta) \subset H_\beta^0$ *and* $\alpha_{\rm HJM} : H_\beta \to H_\beta^0$ *is Fréchet differentiable.*

Proof. Using our previous notation, we can express the HJM drift term (3.2) as

$$\alpha_{\rm HJM} = \sum_{j=1}^d m(\sigma^j, \mathcal{I}\sigma^j) - \sum_{k=1}^e m(-\delta^k, \Psi_k'(-\mathcal{I}\delta^k)).$$

Noting that $H_{\beta'}^0 \subset H_\beta^0$ with isometric embedding by (2.2), we obtain the desired result by using Lemmas 4.1, 4.2, 4.3 and the chain rule for Fréchet differentiable maps. □

4.7. **Assumption.** *We assume there exists a constant* $L > 0$ *such that (3.4), (3.5) are satisfied for all* $h_1, h_2 \in H_\beta$, *and a constant* $M > 0$ *such that (3.8), (3.9) are satisfied for all* $h \in H_\beta$.

4.8. **Proposition.** *Suppose Assumptions 3.1, 4.4, 4.7 are fulfilled. Then, there exists a constant* $\tilde{L} > 0$ *such that (3.12) is satisfied for all* $h_1, h_2 \in H_\beta$.

Proof. The claim follows from [13, Cor. 5.1.2] and [14, Prop. 4.5]. □

4.9. **Remark.** *Note that Assumption 4.4 follows from Assumption 4.7, if we replace (3.9) by the stronger condition*

$$\|\delta^k(h)\|_{\beta'} < \sqrt{\beta'(\beta'-\beta)\tfrac{N}{C}}, \quad k = 1,\ldots,e$$

for all $h \in H_\beta$. *This follows from Lemma 4.1.*

Motivated by ideas from convenient analysis, see [22], we fix a differentiable curve of initial data $\epsilon \mapsto c(\epsilon) \in L^2(\Omega, \mathcal{F}_0, \mathbb{P}; H_\beta)$ with $c(0) = h_0$ and $c'(0) = w$. Let $(r_t)_{t\geq 0}$ be the mild solution for (3.14) on the state space H_β. For $\epsilon \neq 0$ let $(\Delta_t^\epsilon)_{t\geq 0}$ be the mild solution for

$$\begin{cases} d\Delta_t^\epsilon &= (\frac{d}{dx}\Delta_t^\epsilon + \frac{\alpha_{\rm HJM}(r_t + \epsilon\Delta_t^\epsilon) - \alpha_{\rm HJM}(r_t)}{\epsilon})dt + \sum_{j=1}^d \frac{\sigma^j(r_t+\epsilon\Delta_t^\epsilon) - \sigma^j(r_t)}{\epsilon} dW_t^j \\ & \quad + \sum_{k=1}^e \frac{\delta^k(r_{t-} + \epsilon\Delta_{t-}^\epsilon) - \delta^k(r_{t-})}{\epsilon} dX_t^k \\ \Delta_0^\epsilon &= \frac{c(\epsilon)-c(0)}{\epsilon}. \end{cases}$$

Moreover, let the first variation process $((J(r) \bullet w)_t)_{t\geq 0}$ in direction w be the mild solution for

$$\begin{cases} d(J(r)\bullet w)_t &= (\frac{d}{dx}(J(r)\bullet w)_t + D\alpha_{\rm HJM}(r_t) \bullet (J(r)\bullet w)_t)dt \\ & \quad + \sum_{j=1}^d D\sigma^j(r_t) \bullet (J(r)\bullet w)_t dW_t^j \\ & \quad + \sum_{k=1}^e D\delta^k(r_{t-}) \bullet (J(r)\bullet w)_{t-} dX_t^k \\ (J(r)\bullet w)_0 &= w. \end{cases}$$

Here is our result on the regular dependence on initial data for Lévy term structure models.

4.10. **Theorem.** *Suppose Assumptions 3.1, 4.4, 4.5, 4.7 are fulfilled. Then, for every* $T \in \mathbb{R}_+$ *we have*

$$\mathbb{E}\bigg[\sup_{t\in[0,T]} \|(J(r)\bullet w)_t - \Delta_t^\epsilon\|_\beta^2 \bigg] \to 0$$

for $\epsilon \to 0$. Moreover, the map $w \mapsto J(r) \bullet w$ is linear and continuously depending on w in the sense that for every $T \in \mathbb{R}_+$ we have

$$\mathbb{E}\bigg[\sup_{t \in [0,T]} \| (J(r) \bullet w_n)_t - (J(r) \bullet w)_t \|_\beta^2 \bigg] \to 0$$

for variation of the initial value $w_n \to w \in L^2(\Omega, \mathcal{F}_0, \mathbb{P}; H_\beta)$.

Proof. Taking into account Theorem 2.1 and Propositions 4.6, 4.8, the assertion follows from Theorem C.5. □

Appendix A. Stochastic partial differential equations driven by Lévy processes

For convenience of the reader, we provide the crucial results on stochastic partial differential equations (SPDEs) driven by Lévy processes in this appendix.

This section contains the general framework. Appendices B, C are devoted to stability and regular dependence on initial data for stochastic partial differential equations.

Recently, a new approach to stochastic partial differential equations, called the method of the moving frame, was suggested in [15], which allows to reduce a wide class of SPDE problems to SDEs. In what follows, we will refer to the results of this paper. Other reference on stochastic partial differential equations, which also contain regularity results, are [1] and [24]. We remark that the just cited references [15, 1, 24] consider the more general situation, where the SPDE is driven a (possibly infinite dimensional) Wiener process and a compensated Poisson random measure. We also mention the textbooks [5, 26] for SPDEs driven by Wiener processes resp. Lévy processes.

Let H denote a separable Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and associated norm $\| \cdot \|$.

Furthermore, let $(S_t)_{t \geq 0}$ be a C_0-semigroup on H with infinitesimal generator $A : \mathcal{D}(A) \subset H \to H$. We denote by $A^* : \mathcal{D}(A^*) \subset H \to H$ the adjoint operator of A. Recall that the domains $\mathcal{D}(A)$ and $\mathcal{D}(A^*)$ are dense in H, see, e.g., [31, Satz VII.4.6, p. 351].

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ be a filtered probability space satisfying the usual conditions. We denote by $X^1, \dots, X^d$ real-valued, square-integrable, independent Lévy processes, where $d \in \mathbb{N}$ is a positive integer.

We shall now focus on (semi-linear) stochastic partial differential equations

$$\text{(A.1)} \qquad \begin{cases} dr_t &= (Ar_t + \alpha(r_t))dt + \sum_{j=1}^d \sigma^j(r_{t-})dX_t^j \\ r_0 &= h_0 \end{cases}$$

on the separable Hilbert space H with coefficients $\alpha : H \to H$ and $\sigma^j : H \to H$, $j = 1, \dots, d$. The initial condition is an $\mathcal{F}_0$-measurable random variable $h_0 : \Omega \to H$.

A.1. **Definition.** *An adapted, càdlàg H-valued process $(r_t)_{t \geq 0}$ is called a* strong solution *for (A.1) with $r_0 = h_0$ if we have $r_t \in \mathcal{D}(A)$, $t \geq 0$, the relation*

$$\mathbb{P}\bigg(\int_0^t \bigg(\|Ar_s + \alpha(r_s)\| + \sum_{j=1}^d \|\sigma^j(r_s)\|^2 \bigg) ds < \infty \bigg) = 1$$

for all $t \in \mathbb{R}_+$, and

$$r_t = h_0 + \int_0^t (Ar_s + \alpha(r_s))ds + \sum_{j=1}^d \int_0^t \sigma^j(r_{s-})dX_s^j, \quad t \geq 0.$$

A.2. **Definition.** *An adapted, càdlàg H-valued process $(r_t)_{t \geq 0}$ is called a* weak solution *for (A.1) with $r_0 = h_0$ if*

$$\mathbb{P}\bigg(\int_0^t \bigg(\|\alpha(r_s)\| + \sum_{j=1}^d \|\sigma^j(r_s)\|^2 \bigg) ds < \infty \bigg) = 1 \tag{A.2}$$

for all $t \in \mathbb{R}_+$, and for all $\zeta \in \mathcal{D}(A^*)$ we have

$$\langle \zeta, r_t \rangle = \langle \zeta, h_0 \rangle + \int_0^t (\langle A^* \zeta, r_s \rangle + \langle \zeta, \alpha(r_s) \rangle) ds + \sum_{j=1}^d \int_0^t \langle \zeta, \sigma^j(r_{s-}) \rangle dX_s^j, \quad t \geq 0.$$

A.3. **Definition.** *An adapted, càdlàg H-valued process $(r_t)_{t \geq 0}$ is called a* mild solution *for (A.1) with $r_0 = h_0$ if we have (A.2) for all $t \in \mathbb{R}_+$, and*

$$r_t = S_t h_0 + \int_0^t S_{t-s} \alpha(r_s) ds + \sum_{j=1}^d \int_0^t S_{t-s} \sigma^j(r_{s-}) dX_s^j, \quad t \geq 0.$$

By convention, *uniqueness* of solutions for (A.1) is meant up to indistinguishability, that is, for two solutions r^1, r^2 we have $\mathbb{P}(\bigcap_{t \in \mathbb{R}_+} \{r_t^1 = r_t^2\}) = 1$.

For our subsequent investigations, we will impose the following assumption, which allows to apply the method of the moving frame from [15].

A.4. **Assumption.** *There exist another separable Hilbert space $\mathcal{H}$, a C_0-group $(U_t)_{t \in \mathbb{R}}$ on $\mathcal{H}$ and continuous linear operators $\ell \in L(H, \mathcal{H})$, $\pi \in L(\mathcal{H}, H)$ such that the diagram*

$$\begin{array}{ccc} \mathcal{H} & \xrightarrow{U_t} & \mathcal{H} \\ \uparrow \ell & & \downarrow \pi \\ H & \xrightarrow{S_t} & H \end{array}$$

commutes for every $t \in \mathbb{R}_+$, that is

$$\pi U_t \ell h = S_t h \quad \text{for all } t \in \mathbb{R}_+ \text{ and } h \in H.$$

Assumption A.4 is, according to [15, Prop. 8.5], in particular fulfilled if the semigroup $(S_t)_{t \geq 0}$ is pseudo-contractive, that is there exists $\omega \in \mathbb{R}$ such that

$$\|S_t\| \leq e^{\omega t}, \quad t \geq 0.$$

The proof of the preceding statement relies on the Skőkefalvi-Nagy theorem on unitary dilations (see e.g. [30, Thm. I.8.1], or [6, Sec. 7.2]). The idea to use the Skőkefalvi-Nagy theorem on unitary dilations in order to overcome the difficulties arising from stochastic convolutions, which occur when dealing with mild solutions, is due to E. Hausenblas and J. Seidler, see [18] and [17].

Appendix B. Stability of stochastic partial differential equations driven by Lévy processes

In this section, we provide the required stability result.

B.1. **Assumption.** *We assume there exists a constant $L > 0$ such that*

$$\|\alpha(h_1) - \alpha(h_2)\| \leq L \|h_1 - h_2\| \tag{B.1}$$

$$\|\sigma^j(h_1) - \sigma^j(h_2)\| \leq L \|h_1 - h_2\|, \quad j = 1, \dots, d \tag{B.2}$$

$$\|\alpha_n(h_1) - \alpha_n(h_2)\| \leq L \|h_1 - h_2\|, \quad n \in \mathbb{N} \tag{B.3}$$

$$\|\sigma_n^j(h_1) - \sigma_n^j(h_2)\| \leq L \|h_1 - h_2\|, \quad j = 1, \dots, d \text{ and } n \in \mathbb{N} \tag{B.4}$$

for all $h_1, h_2 \in H$.

Let $h_0 \in L^2(\Omega, \mathcal{F}_0, \mathbb{P}; H)$ and for each $n \in \mathbb{N}$ let $h_0^n \in L^2(\Omega, \mathcal{F}_0, \mathbb{P}; H)$ be initial values. Then, there exists a unique càdlàg, adapted, mean square continuous mild and weak solution $(r_t)_{t \geq 0}$ for (A.1), and for each $n \in \mathbb{N}$ there exists a unique càdlàg, adapted, mean square continuous mild and weak solution $(r_t^n)_{t \geq 0}$ for

$$\begin{cases} dr_t^n &= (Ar_t^n + \alpha_n(r_t^n))dt + \sum_{j=1}^d \sigma_n^j(r_{t-}^n)dX_t^j \\ r_0^n &= h_0^n, \end{cases}$$

see [15, Cor. 10.6].

B.2. **Assumption.** *We assume that for all $h \in H_\beta$ we have $\alpha_n(h) \to \alpha(h)$ and $\sigma_n^j(h) \to \sigma^j(h)$, $j = 1, \ldots, d$. Furthermore, we assume that $h_0^n \to h_0$ in $L^2(\Omega, \mathcal{F}_0, \mathbb{P}; H)$.*

B.3. **Lemma.** *Suppose Assumptions B.1, B.2 are fulfilled. Let $(r_t)_{t \geq 0}$ be a predictable, mean square continuous H-valued process. Then, for each $T \in \mathbb{R}_+$ we have*

$$\mathbb{E}\bigg[\int_0^T \| \alpha(r_t) - \alpha_n(r_t) \|^2 dt \bigg] + \sum_{j=1}^d \mathbb{E}\bigg[\int_0^T \| \sigma^j(r_t) - \sigma_n^j(r_t) \|^2 dt \bigg] \to 0$$

as $n \to \infty$.

Proof. By Assumption B.1, for all $h \in H$ we have

$$\begin{aligned} \|\alpha(h)\| &\leq \|\alpha(h) - \alpha(0)\| + \|\alpha(0)\| \leq L\|h\| + \|\alpha(0)\|, \\ \|\alpha_n(h)\| &\leq \|\alpha(h) - \alpha(0)\| + \|\alpha(0)\| \leq L\|h\| + \|\alpha_n(0)\|, \quad n \in \mathbb{N} \end{aligned}$$

Since $\alpha_n(0) \to \alpha(0)$, there exists a constant $K > 0$ such that

$$\|\alpha(h) - \alpha_n(h)\| \leq K(1 + \|h\|), \quad n \in \mathbb{N}$$

for all $h \in H$. We obtain analogous statements for σ^j, $j = 1, \ldots, d$. By the mean square continuity of $(r_t)_{t \geq 0}$ and Lebesgue's theorem, the claim follows. $\square$

B.4. **Theorem.** *Suppose Assumptions A.4, B.1, B.2 are fulfilled. Then, for every $T \in \mathbb{R}_+$ we have*

$$\mathbb{E}\bigg[\sup_{t \in [0,T]} \|r_t - r_t^n\|^2 \bigg] \to 0$$

for $n \to \infty$.

Proof. Taking into account Lemma B.3 and the mean square continuity of the solution process $(r_t)_{t \geq 0}$ for (A.1), the claim follows from [15, Prop. 9.1]. $\square$

Appendix C. Regular dependence on initial data for stochastic partial differential equations driven by Lévy processes

In this section, we provide the required regularity result.

C.1. **Assumption.** *We assume there exists a constant $L > 0$ such that (B.1), (B.2) are satisfied for all $h_1, h_2 \in H$.*

C.2. **Assumption.** *We assume that α and σ^j, $j = 1, \ldots, d$ are Gâteaux differentiable.*

C.3. **Lemma.** *Suppose Assumptions C.1, C.2 are fulfilled. Let $h \in H$ and $\epsilon \neq 0$ be arbitrary. Then we have*

$$\begin{aligned} \bigg\| \frac{\alpha(h + \epsilon v_1) - \alpha(h)}{\epsilon} - \frac{\alpha(h + \epsilon v_2) - \alpha(h)}{\epsilon} \bigg\| &\leq L\|v_1 - v_2\|, \\ \bigg\| \frac{\sigma^j(h + \epsilon v_1) - \sigma^j(h)}{\epsilon} - \frac{\sigma^j(h + \epsilon v_2) - \sigma^j(h)}{\epsilon} \bigg\| &\leq L\|v_1 - v_2\|, \quad j = 1, \ldots, d \end{aligned}$$

for all $v_1, v_2 \in H$. *Moreover, we have*

$$\|D\alpha(h)\| \leq L,$$
$$\|D\sigma^j(h)\| \leq L, \quad j = 1, \dots, d$$

for all $h \in H$.

Proof. The claim follows directly by inspection. □

C.4. **Lemma.** *Suppose Assumptions C.1, C.2 are fulfilled. Let* $(r_t)_{t \geq 0}$ *and* $(v_t)_{t \geq 0}$ *be predictable, mean square continuous processes. Then, for each* $T \in \mathbb{R}_+$ *we have*

$$\mathbb{E}\bigg[\int_0^T \bigg\| \frac{\alpha(r_t + \epsilon v_t) - \alpha(r_t)}{\epsilon} - D\alpha(r_t) \bullet v_t \bigg\|^2 dt\bigg] \to 0,$$
$$\mathbb{E}\bigg[\int_0^T \bigg\| \frac{\sigma^j(r_t + \epsilon v_t) - \sigma^j(r_t)}{\epsilon} - D\sigma^j(r_t) \bullet v_t \bigg\|^2 dt\bigg] \to 0, \quad j = 1, \dots, d$$

for $\epsilon \to 0$.

Proof. By virtue of Lemma C.3, we can argue as in the proof of Lemma B.3. □

Motivated by ideas from convenient analysis, see [22], we fix a differentiable curve of initial data $\epsilon \mapsto c(\epsilon) \in L^2(\Omega, \mathcal{F}_0, \mathbb{P}; H)$ with $c(0) = h_0$ and $c'(0) = w$. Let $(r_t)_{t \geq 0}$ be the mild solution for (A.1). For $\epsilon \neq 0$ let $(\Delta_t^\epsilon)_{t \geq 0}$ be the mild solution for

$$\text{(C.1)} \quad \begin{cases} d\Delta_t^\epsilon &= (A\Delta_t^\epsilon + \frac{\alpha(r_t + \epsilon\Delta_t^\epsilon) - \alpha(r_t)}{\epsilon})dt + \sum_{j=1}^d \frac{\sigma^j(r_{t-} + \epsilon\Delta_{t-}^\epsilon) - \sigma^j(r_{t-})}{\epsilon} dX_t^j \\ \Delta_0^\epsilon &= \frac{c(\epsilon) - c(0)}{\epsilon}. \end{cases}$$

Moreover, let the first variation process $((J(r) \bullet w)_t)_{t \geq 0}$ in direction w be the mild solution for

$$\text{(C.2)} \quad \begin{cases} d(J(r) \bullet w)_t &= (A(J(r) \bullet w)_t + D\alpha(r_t) \bullet (J(r) \bullet w)_t)dt \\ & \quad + \sum_{j=1}^d D\sigma^j(r_{t-}) \bullet (J(r) \bullet w)_{t-} dX_t^j \\ (J(r) \bullet w)_0 &= w. \end{cases}$$

Remark that the stochastic partial differential equations (C.1), (C.2) are no longer of the Markovian type (A.1). The claimed existence and uniqueness result for these equations follows from [15, Thm. 8.6] by taking into account Lemma C.3.

C.5. **Theorem.** *Suppose Assumptions A.4, C.1, C.2 are fulfilled. Then, for every* $T \in \mathbb{R}_+$ *we have*

$$\mathbb{E}\bigg[\sup_{t \in [0,T]} \|(J(r) \bullet w)_t - \Delta_t^\epsilon\|^2\bigg] \to 0$$

for $\epsilon \to 0$. *Moreover, the map* $w \mapsto J(r) \bullet w$ *is linear and continuously depending on* w *in the sense that for every* $T \in \mathbb{R}_+$ *we have*

$$\mathbb{E}\bigg[\sup_{t \in [0,T]} \|(J(r) \bullet w_n)_t - (J(r) \bullet w)_t\|^2\bigg] \to 0$$

for variation of the initial value $w_n \to w \in L^2(\Omega, \mathcal{F}_0, \mathbb{P}; H)$.

Proof. By virtue of Lemma C.4, we can apply [15, Prop. 6.2], whence the claimed result holds true for stochastic differential equations. Using the method of the moving frame, we can transfer the regularity result to stochastic partial differential equations as in [15, Sec. 9]. □

References

[1] Albeverio, S., Mandrekar, V., Rüdiger, B. (2008): Existence of mild solutions for stochastic differential equations and semilinear equations with non Gaussian Lévy noise. *Stochastic Processes and Their Applications*, to appear.

[2] Björk, T., Di Masi, G., Kabanov, Y., Runggaldier, W. (1997): Towards a general theory of bond markets. *Finance and Stochastics* **1**(2), 141–174.

[3] Björk, T., Kabanov, Y., Runggaldier, W. (1997): Bond market structure in the presence of marked point processes. *Mathematical Finance* **7**(2), 211–239.

[4] Carmona, R., Tehranchi, M. (2006): *Interest rate models: an infinite dimensional stochastic analysis perspective.* Berlin: Springer.

[5] Da Prato, G., Zabczyk, J. (1992): Stochastic equations in infinite dimensions. New York: Cambridge University Press.

[6] Davies, E. B. (1976): Quantum theory of open systems. London: Academic Press.

[7] Eberlein, E., Raible, S. (1999): Term structure models driven by general Lévy processes. *Mathematical Finance* **9**(1), 31–53.

[8] Eberlein, E., Özkan, F. (2003): The defaultable Lévy term structure: ratings and restructuring. *Mathematical Finance* **13**, 277–300.

[9] Eberlein, E., Jacod, J., Raible, S. (2005): Lévy term structure models: no-arbitrage and completeness. *Finance and Stochastics* **9**, 67–88.

[10] Eberlein, E., Kluge, W. (2006): Exact pricing formulae for caps and swaptions in a Lévy term structure model. *Journal of Computational Finance* **9**(2), 99–125.

[11] Eberlein, E., Kluge, W. (2006): Valuation of floating range notes in Lévy term structure models. *Mathematical Finance* **16**, 237–254.

[12] Eberlein, E., Kluge, W. (2007): Calibration of Lévy term structure models. In *Advances in Mathematical Finance:* In Honor of Dilip Madan, M. Fu, R. A. Jarrow, J.-Y. Yen, and R. J. Elliott (Eds.), Birkhäuser, pp. 155–180.

[13] Filipović, D. (2001): *Consistency problems for Heath–Jarrow–Morton interest rate models.* Berlin: Springer.

[14] Filipović, D., Tappe, S. (2008): Existence of Lévy term structure models. *Finance and Stochastics* **12**, 83–115.

[15] Filipović, D., Tappe, S., Teichmann, J. (2008): Jump-diffusions in Hilbert spaces: Existence, stability and numerics. Preprint. (`http://arxiv.org/abs/0810.5023`)

[16] Filipović, D., Tappe, S., Teichmann, J. (2009): Term structure models driven by Wiener process and Poisson measures: Existence and positivity. Preprint. (`http://arxiv.org/abs/0905.1413`)

[17] Hausenblas, E., Seidler, J. (2001): A note on maximal inequality for stochastic convolutions. *Czechoslovak Mathematical Journal* **51**(126), 785–790.

[18] Hausenblas, E., Seidler, J. (2008): Stochastic convolutions driven by martingales: Maximal inequalities and exponential integrability. *Stoch. Anal. Appl.* **26**(1), 98–119.

[19] Heath, D., Jarrow, R., Morton, A. (1992): Bond pricing and the term structure of interest rates: a new methodology for contingent claims valuation. *Econometrica* **60**(1), 77–105.

[20] Hyll, M. (2000): Affine term structures and short-rate realizations of forward rate models driven by jump-diffusion processes. In *Essays on the term structure of interest rates* PhD thesis, Stockholm School of Economics.

[21] Jarrow, A., Madan, D. B. (1995): Option pricing using the term structure of interest rates to hedge systematic discontinuities in asset returns. *Mathematical Finance* **5**(4), 311–336.

[22] Kriegl, A., Michor, P. (1997): The convenient setting of Global Analysis, Mathematical Surveys and Monographs **53**, American Mathematical Society, Providence.

[23] Marinelli, C. (2008): Local well-posedness of Musiela's SPDE with Lévy noise. *Mathematical Finance*, to appear.

[24] Marinelli, C., Prévôt, C., Röckner, M. (2008): Regular dependence on initial data for stochastic evolution equations with multiplicative Poisson noise. Preprint. (`http://arxiv.org/abs/0808.1509`)

[25] Musiela, M. (1993): Stochastic PDEs and term structure models. *Journées Internationales de Finance*, IGR-AFFI, La Baule.

[26] Peszat, S., Zabczyk, J. (2007): *Stochastic partial differential equations with Lévy noise.* Cambridge University Press, Cambridge.

[27] Peszat, S., Zabczyk, J. (2007): Heath-Jarrow-Morton-Musiela equation of bond market. Preprint IMPAN 677, Warsaw. (`www.impan.gov.pl/EN/Preprints/index.html`)

[28] Sato, K. (1999): *Lévy processes and infinitely divisible distributions.* Cambridge studies in advanced mathematics, Cambridge.

[29] Shirakawa, H. (1991): Interest rate option pricing with Poisson-Gaussian forward rate curve processes. *Mathematical Finance* **1**(4), 77–94.
[30] Sz.-Nagy, B., Foiaş, C. (1970): Harmonic analysis of operators on Hilbert space. North-Holland, Amsterdam.
[31] Werner, D. (2002): *Funktionalanalysis.* Fourth Edition, Berlin: Springer.

Vienna Institute of Finance, University of Vienna, and Vienna University of Economics and Business Administration, Heiligenstädter Strasse 46-48, A-1190 Wien, Austria; University of Koblenz-Landau, Department of Mathematics, Universitätsstrasse 1, D-56070 Koblenz, Germany

E-mail address: stefan.tappe@vif.ac.at, ruediger@uni-koblenz.de